Polynomdivision

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Die Suche nach ganzzahligen Nullstellen

$$ax^2+bx+c = a(x-x_1)(x-x_2)$$

$$3x^2-21x+30 = 3\cdot(x^2-7x+10) = 3\cdot(x-5)(x-2)$$

Schritte: Den Koeffizienten a ausklammern, Satz von Vieta

Zusammenhang von $a=3, c=30, x_1=5, x_2=2$:

$$3\cdot 5\cdot 2 = 30 \quad \text{, also} \quad a\cdot x_1\cdot x_2 = c \quad \text{bzw.} \quad x_1 x_2 = \frac{c}{a}$$

Polynom 3. Grades:

$$ax^3+bx^2+cx+d = a(x-x_1)(x-x_2)(x-x_3)$$

Zusammenhang a, d, x_1, x_2, x_3:

$$a\cdot x_1\cdot x_2\cdot x_3 = d \quad \text{bzw.} \quad x_1\cdot x_2\cdot x_3 = \frac{d}{a}$$

Trick zum Erraten von Nullstellen:

Suche unter den Teilern der Zahl $\frac{d}{a}$ nach Nullstellen.

Aufgaben:

1. $x^3-10x^2+31x-30 = 0$ $\qquad L=\{2;3;5\}$
2. $x^3+8x^2-5x-84 = 0$ $\qquad L=\{-7;-4;3\}$
3. $2x^4-9x^3-10x^2+27x-10=0$ $\qquad L=\{-2;\frac{1}{2};1;5\}$
4. $2x^4-26x^2+72=0$ $\qquad L=\{\pm 2;\pm 3\}$

Die Polynomdivision

$$\begin{array}{l} 1584 : 12 = 132 \\ \underline{1200} \\ \;\;380 \\ \;\;\underline{360} \\ \;\;\;\;24 \\ \;\;\;\;\underline{24} \\ \;\;\;\;\;0 \end{array}$$

Vergleich mit der Rechnung im Kasten: Ergänzt man die Nullen (hier in blau), so erkennt man, dass es sich um dieselben Zahlen handelt.

$$\begin{array}{l} (1x^3+5x^2+8x+4):(1x+2) = 1x^2+3x+2 \\ \underline{-(1x^3+2x^2)} \\ \qquad 3x^2+8x \\ \qquad \underline{-(3x^2+6x)} \\ \qquad\qquad 2x+4 \\ \qquad\qquad \underline{-(2x+4)} \\ \qquad\qquad\qquad 0 \end{array}$$

Kontrolle:

$$(x^2+3x+2)\cdot(x+2) = x^3+2x^2+3x^2+6x+2x+4$$

$$= x^3+5x^2+8x+4 \quad ✓$$

Aufgaben:

1. a) $(5x^3+21x^2-56x-12):(x+6)=5x^2-9x-2$

$$
\begin{array}{l}
\underline{-(5x^3+30x^2)} \\
\quad -9x^2-56x \\
\quad \underline{-(-9x^2-54x)} \\
\qquad -2x-12 \\
\qquad \underline{-(-2x-12)} \\
\qquad\qquad 0
\end{array}
$$

b) $(2x^3+2x^2-21x+12):(x+4)=2x^2-6x+3$

$$
\begin{array}{l}
\underline{-(2x^2+8x^2)} \\
\quad -6x^2-21x \\
\quad \underline{-(-6x^2-24x)} \\
\qquad 3x+12 \\
\qquad \underline{-(3x+12)} \\
\qquad\qquad 0
\end{array}
$$

c) $(2x^3-7x^2-x+2):(2x-1)=x^2-3x-2$

$$
\begin{array}{l}
\underline{-(2x^3-x^2)} \\
\quad -6x^2-x \\
\quad \underline{-(-6x^2+3x)} \\
\qquad -4x+2 \\
\qquad \underline{-(-4x+2)} \\
\qquad\qquad 0
\end{array}
$$

2. a) $(3x^3-15x^2-36x+108):(x-2)=3x^2-9x-54$ Erraten: $x_1=2$

$$
\begin{array}{l}
\underline{-(3x^3-6x^2)} \\
\quad -9x^2-36x \\
\quad \underline{-(-9x^2+18x)} \\
\qquad -54x+108 \\
\qquad \underline{-(-54x+108)} \\
\qquad\qquad 0
\end{array}
$$

$$
\begin{aligned}
3x^3-15x^2-36x+108 &= (x-2)\cdot(3x^2-9x-54) \\
&= 3\cdot(x-2)\cdot(x^2-3x-18) \qquad \text{(Vieta!)} \\
&= \underline{3(x-2)\cdot(x-6)\cdot(x+3)}
\end{aligned}
$$

b) $(2x^4-9x^3-10x^2+27x-10):(x-1)=2x^3-7x^2-17x+10$; Erraten: $x_1=1$

$$
\begin{array}{l}
\underline{-(2x^4-2x^3)} \\
\quad -7x^3-10x^2 \\
\quad \underline{-(-7x^3+7x^2)} \\
\qquad -17x^2+27x \\
\qquad \underline{-(-17x^2+17x)} \\
\qquad\quad 10x-10 \\
\qquad\quad \underline{-(10x-10)} \\
\qquad\qquad\quad 0
\end{array}
$$

$(2x^3-7x^2-17x+10):(x-5)=2x^2+3x-2$ Erraten: $x_2=5$

$$
\begin{array}{l}
\underline{-(2x^3-10x^2)} \\
\quad 3x^2-17x \\
\quad \underline{-(3x^2-15x)} \\
\qquad -2x+10 \\
\qquad \underline{-(-2x+10)}
\end{array}
$$

0

$$(2x^4-9x^3-10x^2+27x-10) = (x-1)(x-5)\cdot(2x^2+3x-2)$$
$$= 2(x-1)(x-5)(x^2+1{,}5x-1) \qquad \text{(Vieta)}$$
$$= \underline{2(x-1)(x-5)(x+2)(x-0{,}5)}$$

c) $(x^3+19x^2+55x-363):(x+11) = x^2+8x-33$ Erraten: $x_1=-11$

$-(x^3+11x^2)$

$\quad 8x^2+55x$

$\quad -(8x^2+88x)$

$\qquad -33x-363$

$\qquad -(-33x-363)$

$\qquad\qquad 0$

$$(x^3+19x^2+55x-363) = (x+11)\cdot(x^2+8x-33) \qquad \text{(Vieta)}$$
$$= (x+11)\cdot(x-3)(x+11)$$
$$= \underline{(x+11)^2\cdot(x-3)}$$

3. $f(x) = 5x^6-42x^5+82{,}5x^4+70x^3-180x^2$

Extremstellen (nur notw. Bed.):

$$f'(x) = 30x^5-210x^4+330x^3+210x^2-360x$$
$$= 30x\cdot(x^4-7x^3+11x^2+7x-12)$$

$x_1=0$

$x_2=1$ (erraten)

$(x^4-7x^3+11x^2+7x-12):(x-1) = x^3-6x^2+5x+12$

$-(x^4-x^3)$

$\quad -6x^3+11x^2$

$\quad -(-6x^3+6x^2)$

$\qquad 5x^2+7x$

$\qquad -(5x^2-5x)$

$\qquad\quad 12x-12$

$\qquad\quad -(12x-12)$

$\qquad\qquad 0$

$x_3=-1$ (erraten)

$(x^3-6x^2+5x+12):(x+1) = x^2-7x+12$

$= (x-3)(x-4)$ (Vieta)

$\Rightarrow x_4=3,\ x_5=4$

$-(x^3+x^2)$

$\quad -7x^2+5x$

$\quad -(-7x^2-7x)$

$\qquad 12x+12$

$\qquad -(12x+12)$

$\qquad\quad 0$

Die Extremstellen sind $\underline{x_1=0;\ x_2=1;\ x_3=-1;\ x_4=3;\ x_5=4.}$

4. Beispiel: $(x^3-2x^2+x-2):(x^2+1) = x-2$

$-(x^3 \qquad +x)$

$\quad -2x^2+0-2$

$\quad -(-2x^2 \qquad -2)$

$\qquad\qquad 0$

a) $(x^5+x^4-x^3+0x^2+2x+1):(x^2+2x+1)=x^3-x^2+1$

$$\begin{array}{l}
-(x^5+2x^4+x^3)\\
\quad -x^4-2x^3+0x^2\\
\quad -(-x^4-2x^3-x^2)\\
\qquad\quad x^2+2x+1\\
\qquad\quad -(x^2+2x+1)\\
\qquad\qquad\qquad 0
\end{array}$$

b) $(2x^5+3x^4+8x^3+2x^2+3x+8):(x^3+1)=2x^2+3x+8$

$$\begin{array}{l}
-(2x^5 \qquad\qquad +2x^2)\\
\qquad 3x^4+8x^3+0x^2+3x\\
\quad -(3x^4 \qquad\qquad +3x)\\
\qquad\qquad 8x^3+0x^2+0x+8\\
\qquad\quad -(8x^3 \qquad\qquad +8)\\
\qquad\qquad\qquad\qquad 0
\end{array}$$

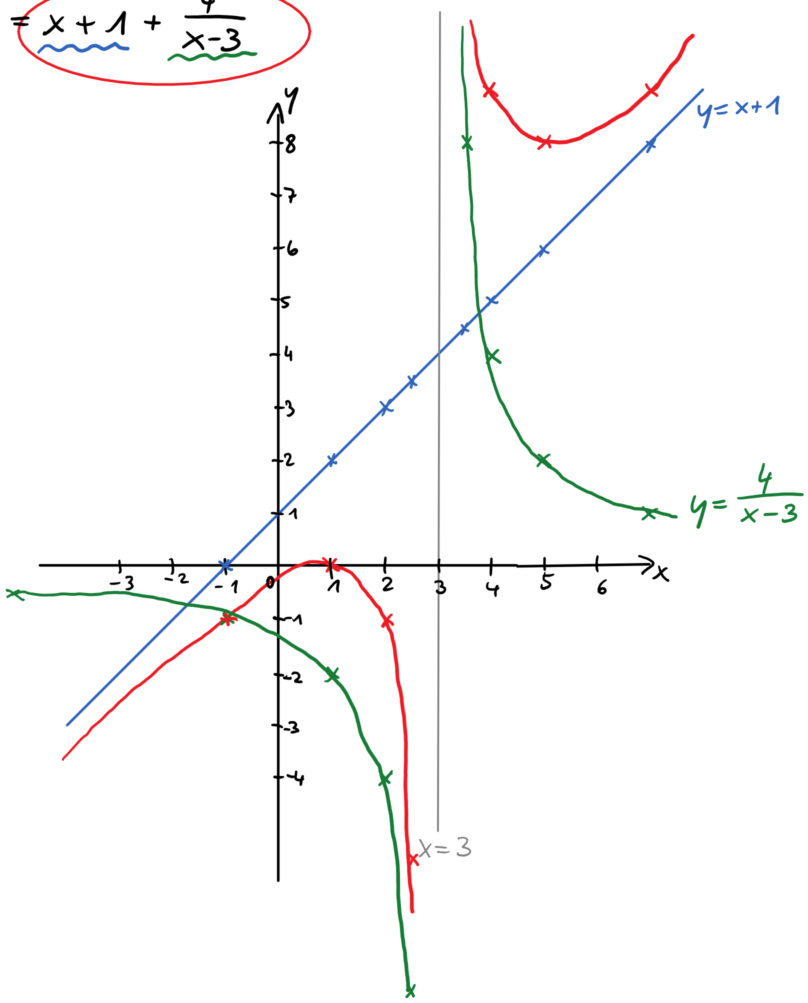
5. Beispiel: $(2x^4+4x^3+8x):(x^3+1)=2x+4+\frac{6x-4}{x^3+1}$

$$\begin{array}{l}
-(2x^4 \qquad +2x)\\
\qquad 4x^3+6x\\
\quad -(4x^3 \quad +4)\\
\qquad\qquad 6x-4
\end{array}$$

a) $f(x)=\frac{x^2-2x+1}{x-3}$

$(x^2-2x+1):(x-3)=x+1+\frac{4}{x-3}$

$$\begin{array}{l}
-(x^2-3x)\\
\qquad x+1\\
\quad -(x-3)\\
\qquad\quad 4
\end{array}$$



b) $f(x) = \dfrac{x^2+x-6}{x+2}$

$(x^2+x-6) : (x+2) = x-1 + \dfrac{-4}{x+2}$

$-(x^2+2x)$

$-x-6$

$-(-x-2)$

-4