Bruchgleichungen

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<u>Erarbeitung</u>: $\frac{2x}{x^2+x} = \frac{4x+2}{x(x+1)} + \frac{2}{x}$ (1)

1. Definitionsmenge: $D = \mathbb{R} \setminus \{-1; 0\}$
2. Hauptnenner: $x(x+1)$

   Vorgehensweise: Alle Nenner in Linearfaktoren zerlegen, von jedem Linearfaktor die höchste auftretende Potenz verwenden, das Produkt davon bilden.

3. Vereinfachen: $\frac{2x}{x^2+x} = \frac{4x+2}{x(x+1)} + \frac{2}{x}$ $\quad | \cdot x(x+1)$

$$\frac{2x \cdot \cancel{x(x+1)}^{1}}{\cancel{x(x+1)}_{1}} = \frac{(4x+2) \cdot \cancel{x(x+1)}^{1}}{\cancel{x(x+1)}_{1}} + \frac{2\cancel{x}^{1}(x+1)}{\cancel{x}_{1}}$$

4. Lösen:

$$2x = 4x+2+2x+2 \quad | -6x$$
$$-4x = 4 \quad | :(-4)$$
$$x = -1$$

5. Vergleich mit D: $-1 \notin D$
6. Lösungsmenge: $\underline{\underline{L = \{\}}}$

<u>Aufgaben</u>:

0. Binomische Formeln:

① $(a+b)^2 = a^2 + 2ab + b^2$

<u>Bsp.</u>: $(x+1)^2 = x^2 + 2x + 1$

$(4x + \frac{1}{2}y)^2 = 16x^2 + xy + \frac{1}{4}y^2$

② $(a-b)^2 = a^2 - 2ab + b^2$

<u>Bsp.</u>: $(x-2)^2 = x^2 - 4x + 4$

$(3y - \sqrt{2}x)^2 = 9y^2 - 6\sqrt{2}xy + 2x^2$

③ $(a-b)(a+b) = a^2 - b^2$

<u>Bsp.</u>: $(x-1)(x+1) = x^2 - 1$

$(5y-4)(5y+4) = 25y^2 - 16$

a) $2x^2 + x = x(2x+1)$

b) $6x^2 + 3x = 3x(2x+1)$

c) $4x - 4 = 4(x-1)$

d) $4x^2 - 4 = 4(x^2-1)$

1. a) $\frac{3}{x} + 2 = x \quad | \cdot x$ $\qquad D = \mathbb{R} \setminus \{0\}$

$$3 + 2x = x^2 \quad | -2x-3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x_1 = 3 \in D; \; x_2 = -1 \in D$$

$\underline{L = \{-1; 3\}}$

b) $\frac{1}{3x^2} - 1 = \frac{1}{6x} \quad |\cdot 6x^2$ $\qquad D = \mathbb{R}\setminus\{0\}$

$2 - 6x^2 = x \quad |-x$

$-6x^2 - x + 2 = 0$

$x_{1/2} = \frac{1 \pm \sqrt{1+48}}{-12} = \frac{1 \pm 7}{12}$

$x_1 = \frac{8}{12} = \frac{2}{3} \in D$

$x_2 = -\frac{6}{12} = -\frac{1}{2} \in D$ $\qquad \underline{L = \{-\frac{1}{2}; \frac{2}{3}\}}$

c) $\frac{1}{x+2} + x = \frac{3x+7}{x+2} \quad |\cdot(x+2)$ $\qquad D = \mathbb{R}\setminus\{-2\}$

$1 + x(x+2) = 3x+7$

$1 + x^2 + 2x = 3x + 7 \quad |-3x-7$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x_1 = 3 \in D \;;\; x_2 = -2 \notin D$ $\qquad \underline{L = \{3\}}$

d) $\frac{2x+1}{3} + \frac{10}{2x+1} = 4 \quad |\cdot 3(2x+1)$ $\qquad D = \mathbb{R}\setminus\{-\frac{1}{2}\}$

$(2x+1)^2 + 30 = 12(2x+1)$

$4x^2 + 4x + 1 + 30 = 24x + 12 \quad |-24x-12$

$4x^2 - 20x + 19 = 0$

$x_{1/2} = \frac{20 \pm \sqrt{400-304}}{8} = \frac{20 \pm 4\sqrt{6}}{8}$

$x_1 = \frac{5}{2} + \frac{1}{2}\sqrt{6} \in D$

$x_2 = \frac{5}{2} - \frac{1}{2}\sqrt{6} \in D$ $\qquad \underline{L = \{\frac{5}{2} - \frac{1}{2}\sqrt{6}; \frac{5}{2} + \frac{1}{2}\sqrt{6}\}}$

2a) $\frac{3}{x+1} + \frac{4}{x-2} = \frac{3}{2x+2} \quad |\cdot 2(x+1)(x-2)$ $\qquad D = \mathbb{R}\setminus\{-1; 2\}$

$6(x-2) + 8(x+1) = 3(x-2)$

$6x - 12 + 8x + 8 = 3x - 6 \quad |-3x+4$

$11x = -2 \quad |:11$

$x = -\frac{2}{11} \in D$ $\qquad \underline{L = \{-\frac{2}{11}\}}$

b) $\frac{x}{x-1} + \frac{2}{x-2} = \frac{1}{x-1} + \frac{x}{x-2} \quad |\cdot(x-1)(x-2)$ $\qquad D = \mathbb{R}\setminus\{1; 2\}$

$x(x-2) + 2(x-1) = x - 2 + x(x-1)$

$x^2 - 2x + 2x - 2 = x - 2 + x^2 - x \quad |+2$

$0 = 0$ wahre Aussage für alle $x \in D$

$\underline{L = D = \mathbb{R}\setminus\{1; 2\}}$

c) $\frac{x}{2x-3} - \frac{1}{2x} = \frac{3}{4x-6}$ $\quad | \cdot 2x(2x-3)$ $\quad D = \mathbb{R} \setminus \{0; \frac{3}{2}\}$

$x \cdot 2x - 1 \cdot (2x-3) = 3 \cdot x$

$2x^2 - 2x + 3 = 3x$ $\quad | -3x$

$2x^2 - 5x + 3 = 0$

$x_{1/2} = \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm 1}{4}$

$x_1 = \frac{3}{2} \notin D \; ; \; x_2 = 1 \in D$ $\quad \underline{L = \{1\}}$

d) $\frac{36}{x+6} - 36 = \frac{36}{x-6}$ $\quad | \cdot (x+6)(x-6)$ $\quad D = \mathbb{R} \setminus \{-6; +6\}$

$36(x-6) - 36(x^2-36) = 36(x+6)$ $\quad | : 36$

$x - 6 - (x^2 - 36) = x + 6$ $\quad | -x - 6 + x^2$

$24 = x^2$

$x_{1/2} = \pm\sqrt{24} = \pm 2\sqrt{6} \in D$ $\quad \underline{L = \{\pm 2\sqrt{6}\}}$

3 a) $\frac{3}{x-4} - \frac{24}{x^2-16} = \frac{3}{x+4} - x^2 + 16$ $\quad | \cdot (x^2-16)$ $\quad D = \mathbb{R} \setminus \{-4; 4\}$

$3(x+4) - 24 = 3(x-4) - (x^2-16)(x^2-16)$

$3x + 12 - 24 = 3x - 12 - (x^4 - 256)$ $\quad | -3x + 12$

$0 = -x^4 + 256$ $\quad | +x^4$

$x^4 = 256$

$x_{1/2} = \pm 4 \quad \notin D$ $\quad \underline{L = \{\}}$

b) $\frac{7(x-5)^2}{6x^2-6} = \frac{5x-1}{3x+3} - \frac{3x-2}{6x-6}$ $\quad | \cdot 6(x-1)(x+1)$ $\quad D = \mathbb{R} \setminus \{1; -1\}$

$7(x-5)^2 = (5x-1) \cdot 2 \cdot (x-1) - (3x-2)(x+1)$

$7(x^2 - 10x + 25) = 10x^2 - 10x - 2x + 2 - 3x^2 - 3x + 2x + 2$

$7x^2 - 70x + 175 = 7x^2 - 13x + 4$ $\quad | -7x^2 + 13x - 175$

$-57x = -171$ $\quad | : (-57)$

$x = 3 \quad \in D$ $\quad \underline{L = \{3\}}$

c) $\frac{3x+2}{x-2} = \frac{x+2}{3x-2}$ $\quad | \cdot (x-2)(3x-2)$ $\quad D = \mathbb{R} \setminus \{2; \frac{2}{3}\}$

$(3x+2)(3x-2) = (x+2)(x-2)$

$9x^2 - 4 = x^2 - 4$ $\quad | +4 - x^2$

$8x^2 = 0$

$x = 0 \quad \in D$ $\quad \underline{L = \{0\}}$

d) $\dfrac{5x+1}{x+2} = 3 + \dfrac{2x^2+3x-8}{x^2+4x+4} \qquad |\cdot(x+2)^2 \qquad D = \mathbb{R}\setminus\{-2\}$

$(5x+1)(x+2) = 3(x+2)^2 + 2x^2+3x-8$

$5x^2+10x+x+2 = 3x^2+12x+12+2x^2+3x-8 \qquad |-5x^2-15x-2$

$-4x = 2 \qquad |:(-4)$

$x = -\frac{1}{2} \in D \qquad \underline{L = \{-\frac{1}{2}\}}$

4a) $\dfrac{x}{a} - \dfrac{a}{x} = \dfrac{3}{2} \qquad |\cdot ax \qquad D = \mathbb{R}\setminus\{0\}$

$x^2 - a^2 = \frac{3}{2}ax \qquad |-\frac{3}{2}ax \qquad a \in \mathbb{R}\setminus\{0\}$

$x^2 - \frac{3}{2}ax - a^2 = 0$

$x_{1/2} = \dfrac{\frac{3}{2}a \pm \sqrt{\frac{9a^2}{4} + 4a^2}}{2} = \frac{3}{4}a \pm \frac{1}{2}\sqrt{\frac{25}{4}a^2}$

$x_{1/2} = \frac{3}{4}a \pm \frac{5}{4}|a|$

Falls $a > 0$: $x_{1/2} = \frac{3}{4}a \pm \frac{5}{4}a$

$x_1 = 2a \quad ; \quad x_2 = -\frac{1}{2}a$

Falls $a < 0$: $x_{1/2} = \frac{3}{4}a \pm \frac{5}{4}(-a)$

$x_1 = -\frac{1}{2}a \quad ; \quad x_2 = 2a \qquad \underline{L = \{-\frac{1}{2}a; 2a \text{ mit } a \neq 0\}}$

b) $\dfrac{x+a}{x-a} - \dfrac{x-a}{x+a} = \dfrac{8a^2}{x^2-a^2} \qquad |\cdot(x^2-a^2) \qquad x \neq a,\ x \neq -a \text{ mit } a \in \mathbb{R}$

$(x+a)(x+a) - (x-a)(x-a) = 8a^2$

$x^2+2ax+a^2-x^2+2ax-a^2 = 8a^2$

$4ax = 8a^2$

1. Fall: $a = 0$: $0 = 0 \qquad \underline{L_0 = \mathbb{R}\setminus\{0\}}$

2. Fall: $a \neq 0$: $x = 2a \qquad \underline{L_a = \{2a \text{ mit } a \neq 0\}}$