Lineare Substitution, logarithmische Integration

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Erarbeitung 1

Funktion f	$f(x)=\sin(2x)$	$f(x)=(2x-4)^3$	$f(x)=g(mx+c)$
Ableitung f'	$f'(x)=2\cos(2x)$	$f'(x)=3\cdot 2(2x-4)^2$	$f'(x)=m\cdot g'(mx+c)$
Stammf. F	$F(x)=-\frac{1}{2}\cos(2x)$	$F(x)=\frac{1}{2}\cdot\frac{1}{4}(2x-4)^4$	$F(x)=\frac{1}{m}\cdot G(mx+c)$
Integral	$\int_0^{\frac{\pi}{4}}\sin(2x)\,dx$ $=\left[-\frac{1}{2}\cos(2x)\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{2}\cdot 0+\frac{1}{2}\cdot 1=\frac{1}{2}$	$\int_2^3 (2x-4)^3\,dx$ $=\left[\frac{1}{8}(2x-4)^4\right]_2^3$ $=\frac{1}{8}\cdot 16-\frac{1}{8}\cdot 0=2$	$\int_a^b g(mx+c)\,dx$ $=\left[\frac{1}{m}G(mx+c)\right]_a^b$

Erarbeitung 2

Funktion f	$f(x)=\ln(x^2+3x)$	$f(x)=\ln(\sin(2x))$	$f(x)=\ln(g(x))$
Ableitung f'	$f'(x)=\frac{1}{x^2+3x}\cdot(2x+3)$ $=\frac{2x+3}{x^2+3x}$	$f'(x)=\frac{1}{\sin(2x)}\cdot 2\cos(2x)$ $=\frac{2\cos(2x)}{\sin(2x)}$	$f'(x)=\frac{1}{g(x)}\cdot g'(x)$ $=\frac{g'(x)}{g(x)}$

Funktion f	$f(x)=\frac{2x}{1+x^2}$	$f(x)=\frac{6e^{2x}}{5+3e^{2x}}$	$f(x)=\frac{g'(x)}{g(x)}$
Stammf F	$F(x)=\ln(1+x^2)$	$F(x)=\ln(5+3e^{2x})$	$F(x)=\ln(g(x))$

Vermutung: Für $f(x)=\frac{1}{x}$ mit $x<0$ gilt $F(x)=\ln(|x|)$.

Prüfen: $F(x)=\ln(-x)$ für $x<0$

$$F'(x)=\frac{1}{-x}\cdot(-1)=\frac{1}{x}=f(x) \quad ✓$$

Verbesserte Regel: Die Stammfunktion der Funktion f mit $f(x)=\frac{g'(x)}{g(x)}$ ist $\boxed{F(x)=\ln(|g(x)|)}$, falls $g(x)\neq 0$.

Aufgaben

1. a) $\int_0^1 3e^{2x-1}\,dx = \left[3\cdot\frac{1}{2}e^{2x-1}\right]_0^1 = \frac{3}{2}e^1 - \frac{3}{2}e^{-1} = \frac{3e}{2} - \frac{3}{2e}$

b) $\int_1^2 (3-2x)^2\,dx = \left[-\frac{1}{2}\cdot\frac{1}{3}(3-2x)^3\right]_1^2 = -\frac{1}{6}\cdot(-1)^3 + \frac{1}{6}\cdot 1^3 = \frac{1}{3}$

c) $\int_0^2 \sqrt{4x+1}\,dx = \left[\frac{1}{4}\cdot\frac{2}{3}(4x+1)^{\frac{3}{2}}\right]_0^2 = \frac{1}{6}\cdot 9^{\frac{3}{2}} - \frac{1}{6}\cdot 1^{\frac{3}{2}} = \frac{27}{6} - \frac{1}{6} = \frac{13}{3}$

d) $\int_0^3 \frac{6}{2x+5}\,dx = \left[6\cdot\frac{1}{2}\ln(2x+5)\right]_0^3 = 3\ln 11 - 3\ln 5$

e) $\int_{-\frac{1}{2}}^2 3\cdot\cos(\pi x)\,dx = \left[\frac{3}{\pi}\sin(\pi x)\right]_{-\frac{1}{2}}^2 = \frac{3}{\pi}\cdot 0 - \frac{3}{\pi}(-1) = \frac{3}{\pi}$

f) $\int_4^{10} \ln\left(\frac{1}{3}x - \frac{1}{3}\right)dx = \left[3\cdot\left(\left(\frac{1}{3}x-\frac{1}{3}\right)\cdot\ln\left(\frac{1}{3}x-\frac{1}{3}\right) - \left(\frac{1}{3}x-\frac{1}{3}\right)\right)\right]_4^{10}$

$= \left[(x-1)\cdot\ln\left(\frac{1}{3}x-\frac{1}{3}\right) - (x-1)\right]_4^{10}$

$= (9\cdot\ln 3 - 9) - (3\cdot\ln 1 - 3)$

$= 9\cdot\ln 3 - 6$

2. a) $\int_0^1 \frac{2e^x}{2e^x+1}\,dx = \left[\ln(2e^x+1)\right]_0^1 = \ln(2e+1) - \ln 3$

b) $\int_1^2 \frac{2x+2}{x^2+2x+3}\,dx = \left[\ln(x^2+2x+3)\right]_1^2 = \ln 11 - \ln 6$

c) $\int_{e^2}^{e^4} \frac{1}{x\cdot\ln(x)}\,dx = \int_{e^2}^{e^4} \frac{\frac{1}{x}}{\ln(x)}\,dx = \left[\ln(\ln(x))\right]_{e^2}^{e^4} = \ln 4 - \ln 2$

3. a) $\int_0^4 \frac{x}{x^2+4}\,dx = \int_0^4 \frac{1}{2}\cdot\frac{2x}{x^2+4}\,dx = \left[\frac{1}{2}\ln(x^2+4)\right]_0^4 = \frac{1}{2}\ln 20 - \frac{1}{2}\ln 4$

b) $\int_1^2 \frac{x^2}{1-8x^3}\,dx = \int_1^2 -\frac{1}{24}\cdot\frac{-24x^2}{1-8x^3}\,dx = \left[-\frac{1}{24}\ln\left(|1-8x^3|\right)\right]_1^2$

$= -\frac{1}{24}\cdot\ln|-63| + \frac{1}{24}\cdot\ln|-7| = -\frac{1}{24}\ln 63 + \frac{1}{24}\ln(7)$

c) $\int_0^{\frac{1}{3}} \frac{-\sin(\pi x)}{\cos(\pi x)}\,dx = \int_0^{\frac{1}{3}} \frac{1}{\pi}\cdot\frac{-\pi\sin(\pi x)}{\cos(\pi x)}\,dx = \left[\frac{1}{\pi}\ln(\cos(\pi x))\right]_0^{\frac{1}{3}}$

$= \frac{1}{\pi}\ln\left(\frac{1}{2}\right) - \frac{1}{\pi}\ln 1 = \frac{1}{\pi}\ln\left(\frac{1}{2}\right)$

4a) zu 2c) $\ldots = \ln 4 - \ln 2 = \ln 2^2 - \ln 2 = 2\ln 2 - \ln 2 = \ln 2$

zu 3a) $\ldots = \frac{1}{2}\ln 20 - \frac{1}{2}\ln 4 = \frac{1}{2}(\ln 5 + \ln 4 - \ln 4) = \frac{1}{2}\ln 5$

zu 3b) $\ldots = \frac{1}{24}\cdot(-\ln(3^2\cdot 7) + \ln 7) = \frac{1}{24}\cdot(-2\ln 3) = -\frac{1}{12}\ln 3$

zu 3c) $\ldots = \frac{1}{\pi}\ln\frac{1}{2} = \frac{1}{\pi}(\ln 1 - \ln 2) = -\frac{\ln 2}{\pi}$

b) $\int_{\ln 2}^{\ln 3} \frac{e^x - e^{-x}}{2e^x + 2e^{-x}}\,dx = \int_{\ln 2}^{\ln 3} \frac{1}{2}\cdot\frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \left[\frac{1}{2}\ln(e^x + e^{-x})\right]_{\ln 2}^{\ln 3}$

$= \frac{1}{2}\ln\left(3+\frac{1}{3}\right) - \frac{1}{2}\ln\left(2+\frac{1}{2}\right) = \frac{1}{2}\ln\left(\frac{10}{3}\right) - \frac{1}{2}\ln\left(\frac{5}{2}\right)$

$= \frac{1}{2}\ln\left(\frac{10}{3}\cdot\frac{2}{5}\right) = \frac{1}{2}\ln\left(\frac{4}{3}\right) = \ln 2 - \frac{1}{2}\ln 3$